Version 1

 $\ensuremath{\textbf{Problem 1}}$ Consider the following system :

$$\begin{cases} x + y - z = k \\ 2x + 3y + kz = 3k \\ x + ky + 3z = 2k \end{cases}$$

$$\begin{bmatrix} 1 & 1 & -1 & k \\ 2 & 3 & k & 1 & 3k \\ 1 & k & 3 & \frac{1}{2} & 2k \end{bmatrix} \mathbf{r}_{3} - \mathbf{r}_{1} - \mathbf{r}_{5} \mathbf{r}_{5} \begin{bmatrix} 1 & 1 & -1 & k \\ 0 & 1 & k+2 & k \\ 0 & k-1 & 4 & k \end{bmatrix} \mathbf{r}_{3} - (k-1)\mathbf{r}_{2} - \mathbf{r}_{5}$$

$$\begin{bmatrix} 1 & 1 & -1 & k \\ 0 & 1 & k+2 & k \\ 0 & 0 & (k+2)(k+2) \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & k \\ 0 & 1 & k+2 & k \\ 0 & 0 & (k-2)(k+3) \end{bmatrix} = \mathbf{A}\mathbf{E}$$

1. For which values of k does the system have no solutions?

2. For which values of k does the system have exactly one solution ?

к ≠ 2,-3 (First 3 columns of AE here pivots 1.e no free veriables) no bad rows

3. For which values of k does the system have infinitely many solutions ?

NAME (First,Last) :

Problem 2 Let A be the 4×4 matrix with columns c_1, c_2, c_3, c_4 . The matrix $B = \begin{pmatrix} \begin{vmatrix} & & & \\ & & & \\ c_1 & c_2 & c_3 & c_4 & c \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\$

(that is *B* is the 4 × 5 matrix that consists of *A* plus an additional fifth column $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$)

reduces to $\begin{pmatrix} 1 & 1 & -1 & 2 & a \\ 0 & 1 & -3 & -2 & b - a \\ 0 & 0 & 1 & 2 & c - b + a \\ 0 & 0 & 0 & 0 & d - a + 2b \end{pmatrix}$

1. Are c_1, c_2, c_3 , and c_4 linearly independent? Justify your answer.

C is reduced to echelon form and only has 3 pivots, there is no pivot in column 4 30 no Ci (2 (3 (4 are not linearly independent

2. Are c_1, c_2, c_4 linearly independent? Justify your answer.

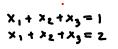
yes looking at
$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$
 there is a pivet in every column so $c_1 c_2 c_4$ are independent.

3. Give an example of a vector $b \in \mathbb{R}^4$ that it is not in span (c_1, c_2, c_3, c_4) , or explain why this is not possible. If you give an example, you need to justify why your example works.

we want a vector
$$\begin{pmatrix} e \\ b \\ c \\ d \end{pmatrix}$$
 with $d-a+ib=0$
For example $b = \begin{bmatrix} o \\ i \\ i \\ d \end{pmatrix}$ would work.

Problem 3 This problem has three unrelated parts.

1. Give an example of a linear system with two equations and three variables that has no solutions, or explain why this is not possible.



2. Give an example of a 3x3 matrix A that has linearly independent columns and can be reduced (by performing a sequence of elementary operations) to a matrix B that has linearly dependent columns, or explain why this is not possible.

Since the columns of A are linearly independent Ax=0 has only the solution $x = \overline{0}^2$, so Bx = 0 also has only the solution $x = \overline{0}^2$, because performing elementary operations transforms a system into a different system with the same solutions. Since $Bx = \overline{0}^2$ only has the triviel solution, the columns of B are also independent. Ax= 0 MPOSSI ble

3. Give an example of three non zero vectors u_1, u_2, u_3 in \mathbb{R}^3 that are linearly dependent, but u_1 is not in span (u_2, u_3) , or explain why this is not possible.

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